ATKINSON SCIENCE

THEORY GUIDE

Vapour Resistance Units Converter Web Application

Keith Atkinson

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Atkinson Science welcomes your comments on this Theory Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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1 Vapour resistivity and vapour resistance

You can find the Atkinson Science Vapour Resistance Units Converter web application at the web address https://atkinsonscience.co.uk/WebApps/Construction/VapResUnitsConverter.aspx. There is a user guide that you can download at the same address.

The vapour resistivity ζ [MN s g^{-1} m^{-1}] is a material property, independent of the thickness of the material.

The vapour resistance z [MN s g^{-1}] applies to a given thickness of material, say 50 mm or 100 mm.

If a material has vapour resistivity $\zeta_{Mat} = 50$ MN s g^{-1} m^{-1} and thickness $L_{Mat} = 40$ mm, then the vapour resistance z_{Mat} of the material is

$$z_{Mat} = \zeta_{Mat} \times L_{Mat} = 50 \times 0.04 = 2 \text{ MN s g}^{-1}$$

There are other measures of the resistance to the flow of vapour through a material. Some are material properties, like the vapour resistivity, while others apply to a given thickness of material, like the vapour resistance.

2 Equivalent air layer thickness (S_d)

The equivalent air layer thickness S_d [m] is the thickness of air in metres that has the same vapour resistance as the material, i.e.

$$z_{Air} = z_{Mat}$$

or

$$\zeta_{Air} \times S_d = \zeta_{Mat} \times L_{Mat}$$

so

$$S_d = \frac{\zeta_{Mat} \times L_{Mat}}{\zeta_{Air}}$$

In the United Kingdom the vapour resistivity of air ζ_{Air} is taken to be 5 MN s g^{-1} m^{-1}.

If a material has vapour resistivity $\zeta_{Mat} = 600$ MN s g^{-1} m^{-1} and thickness $L_{Mat} = 100$ mm, then the equivalent air layer thickness S_d is

$$S_d = \frac{600 \text{ MN s g}^{-1} \text{ m}^{-1} \ \times \ 0.1 \text{ m}}{5 \text{ MN s g}^{-1} \text{ m}^{-1}} = 12 \text{ m}$$

Like the vapour resistance, the equivalent air layer thickness applies to a given thickness of material. In the preceding example the thickness of the material is 100 mm.

3 US perm

The *US permeance* or $US\ perm$ is a measure of the permeability of the material to the flow of vapour, rather than a measure of resistance. The $US\ perm$ is therefore inversely proportional to the vapour resistance. Like the vapour resistance, the $US\ perm$ applies to a given thickness of material.

The $US\ perm$ has the units:

$$US\ perm \equiv \frac{\text{grain}}{\text{hr ft}^2\ \text{inHg}}$$

To determine the $US\ perm$ from the vapour resistance z [MN s g^{-1}], we need the following conversion factors:

$$\frac{\text{g}}{\text{grain}} = 0.06479891$$

$$\frac{\text{s}}{\text{hr}} = 3600$$

$$\frac{\text{m}^2}{\text{ft}^2} = 0.09290304$$

$$\frac{\text{MPa}}{\text{inHg}} = \frac{\text{MN}}{\text{inHg} \times \text{m}^2} = 0.0033863816$$

We can write

$$US\ perm \equiv \frac{\text{grain}}{\text{hr ft}^2\ \text{inHg}} \equiv \frac{\text{grain}}{\text{g}} \times \frac{\text{s}}{\text{hr}} \times \frac{\text{m}^2}{\text{ft}^2} \times \frac{\text{MN}}{\text{inHg} \times \text{m}^2} \times \frac{\text{g}}{\text{s MN}}$$

$$\equiv \frac{1}{0.0647981} \times 3600 \times 0.09290304 \times 0.0033863816 \times \frac{1}{z}$$

so

$$US\ perm \equiv \frac{17.478358}{z}$$

If a material has vapour resistivity $\zeta_{Mat} = 600$ MN s g^{-1} m^{-1} and thickness $L_{Mat} = 100$ mm, then the $US\ perm$ is

$$US\ perm = \frac{17.478358}{z_{Mat}} = \frac{17.478358}{\zeta_{Mat} \times L_{Mat}} = \frac{17.478358}{600 \times 0.1}$$

$$= 0.291306\ \text{grain hr}^{-1}\ \text{ft}^{-2}\ \text{inHG}^{-1}$$

4 SI perm

The $SI\ perm$ or $metric\ perm$ is a metric form of the $US\ perm$ and has the units:

$$SI\ perm \equiv \frac{\text{ng}}{\text{s m}^2\ \text{Pa}}$$

To determine the $SI\ perm$ from the vapour resistance z [MN s g^{-1}], we need the following conversion factors:

$$\frac{\text{ng}}{\text{g}} = 10^9$$

$$\frac{\text{MN}}{\text{N}} = 10^{-6}$$

We can write

$$SI\ perm \equiv \frac{\text{ng}}{\text{s m}^2\ \text{Pa}} \equiv \frac{\text{ng}}{\text{s N}} \equiv \frac{\text{ng}}{\text{g}} \times \frac{\text{MN}}{\text{N}} \times \frac{\text{g}}{\text{s MN}}$$

$$\equiv 10^9 \times 10^{-6} \times \frac{1}{z}$$

so

$$SI\ perm \equiv \frac{1000}{z}$$

If a material has vapour resistivity $\zeta_{Mat} = 600$ MN s g^{-1} m^{-1} and thickness $L_{Mat} = 100$ mm, then the $SI\ perm$ is

$$SI\ perm = \frac{1000}{z_{Mat}} = \frac{1000}{\zeta_{Mat} \times L_{Mat}} = \frac{1000}{600 \times 0.1}$$

$$= 16.6666\ \text{ng s}^{-1}\ \text{m}^{-2}\ \text{Pa}^{-1}$$

5 Water vapour resistance factor (*μ*)

The *water vapour resistance factor* μ [] is the ratio of the vapour resistivity of the material ζ_{Mat} to the vapour resistivity of air ζ_{Air}:

$$\mu = \frac{\zeta_{Mat}}{\zeta_{Air}}$$

In the United Kingdom the vapour resistivity of air ζ_{Air} is taken to be 5 MN s g^{-1} m^{-1}, so

$$\mu = \frac{\zeta_{Mat}}{5\ \mathrm{MN\ s\ g^{-1}\ m^{-1}}}$$

Like the vapour resistivity, μ is a material property (independent of the thickness of the material).

If a material has vapour resistivity $\zeta_{Mat} = 600$ MN s g^{-1} m^{-1}, then the water vapour resistance factor is

$$\mu = \frac{\zeta_{Mat}}{5\ \mathrm{MN\ s\ g^{-1}\ m^{-1}}} = \frac{600\ \mathrm{MN\ s\ g^{-1}\ m^{-1}}}{5\ \mathrm{MN\ s\ g^{-1}\ m^{-1}}} = 120$$

6 Kinematic diffusivity

6.1 Relation to vapour resistivity

Suppose we have a species A diffusing from one side of a species B to the other. The one-dimensional form of *Fick's first law of diffusion* is

$$m'_A = -\rho D \frac{d\, mf_A}{dx} \quad [\mathrm{kg\ s^{-1}\ m^{-2}}] \qquad (6.1)$$

where m'_A [kg s^{-1} m^{-2}] is the mass flux (mass flow rate per unit area) of species A, ρ [kg m^{-3}] is the density of the mixture, D [m^2 s^{-1}] is the kinematic diffusivity of species A in species B, mf_A [] is the mass fraction of A in the mixture, and x [m] is the direction of diffusion.

Most building materials have open pores containing moist air. If there is a difference in vapour concentration across the material, then vapour will diffuse through the air in the direction from high concentration to low concentration. The mass flux of vapour is usually modelled by

$$m_v = -\frac{1}{10^6 \zeta} \frac{dp_v}{dx} \quad [\mathrm{g\ s^{-1}\ m^{-2}}] \qquad (6.2)$$

where m_v [g s^{-1} m^{-2}] is the mass flux (mass flow rate per unit area) of vapour, ζ [MN s g^{-1} m^{-1}] is the vapour resistivity, p_v [Pa] is the pressure of the vapour in the mixture, and x [m] is the direction of diffusion. Dividing the right-hand side by 1000 gives the mass flux m'_v in [kg s^{-1} m^{-2}]:

$$m'_v = -\frac{1}{10^9 \times \zeta} \frac{dp_v}{dx} \quad [\mathrm{kg\ s^{-1}\ m^{-2}}] \qquad (6.3)$$

Multiplying the top and bottom of (6.3) by the pressure of the air in the mixture p_a [Pa] gives

$$m'_v = -\frac{p_a}{10^9 \times \zeta} \frac{d}{dx}\left(\frac{p_v}{p_a}\right) \qquad (6.4)$$

The moisture content ω [] is related to the pressure ratio p_v/p_a by

$$\omega = 0.62196 \frac{p_v}{p_a} \qquad (6.5)$$

Substituting (6.5) into (6.4) gives

$$m'_v = -\frac{p_a}{0.62196 \times 10^9 \times \zeta} \frac{d\omega}{dx} \qquad (6.6)$$

If we approximate the moisture content ω by the mass fraction of vapour mf_v and the pressure of the air in the mixture p_a [Pa] by the atmospheric pressure $p = p_a + p_v$ [Pa], then Eq. (6.6) becomes

$$m'_v = -\frac{p}{0.62196 \times 10^9 \times \zeta} \frac{dmf_v}{dx} \qquad (6.7)$$

We can equate the right-hand sides of Eqs. (6.1) and (6.7) to obtain

$$\rho D = \frac{p}{0.62196 \times 10^9 \times \zeta} \qquad (6.8)$$

This gives the kinematic diffusivity D [m² s⁻¹] in terms of the vapour resistivity ζ [MN s g⁻¹ m⁻¹]:

$$D = \frac{p}{0.62196 \times 10^9 \times \rho\zeta} \qquad (6.9)$$

We can take the atmospheric pressure p to be 101325 Pa. The density of dry air at 20°C is 1.206 [kg m⁻³]. Substituting these values into (6.9) gives

$$D = \frac{135.08 \times 10^{-6}}{\zeta} \ [\mathrm{m^2 s^{-1}}] \qquad (6.10)$$

6.2 Dimensions

We can check that the dimensions of the two sides of Eq. (6.9) are the same. The dimensions of force are:

$$F = Mass \ \times \ Acceleration \equiv [M]\left[\frac{L}{T^2}\right] = \left[\frac{ML}{T^2}\right]$$

The dimensions of pressure are therefore

$$p = \frac{Force}{Area} \equiv \left[\frac{ML}{T^2}\right]\left[\frac{1}{L^2}\right] = \left[\frac{M}{LT^2}\right]$$

The dimensions of density are

$$\rho[\mathrm{kg\ m^{-3}}] \equiv \left[\frac{M}{L^3}\right]$$

The dimensions of the vapour resistivity are

$$\zeta\ [\mathrm{MN\ s\ g^{-1}\ m^{-1}}] \equiv \left[\frac{ML}{T^2}\right][T]\left[\frac{1}{M}\right]\left[\frac{1}{L}\right] = \left[\frac{1}{T}\right]$$

The dimensions of the right-hand side of Eq. (6.9) are therefore

$$\frac{p}{0.62196 \times 10^9 \times \rho\zeta} \equiv \left[\frac{M}{LT^2}\right]\left[\frac{L^3}{M}\right][T] = \left[\frac{L^2}{T}\right]$$

as expected.

6.3 Computational fluid dynamics

In a computational fluid dynamics (CFD) program the diffusion of a conserved scalar ϕ [kg] in a fluid or solid is determined by solving the equation

$$\nabla . \left(\rho D_{\phi} \nabla m f_{\phi}\right) = 0 \qquad (6.11)$$

where ρ [kg m^{-3}] is the density of the fluid or solid, mf_{ϕ} [] is the mass fraction of ϕ (mass of ϕ per unit mass of fluid or solid), and D_{ϕ} [m^2 s^{-1}] is the kinematic diffusivity of ϕ in the fluid or solid. Note that in a building structure, ρ is not constant but varies from one solid material to another.

In vapour diffusion through building materials the vapour diffuses through the pockets of stationary air in the solid material. The density ρ in (6.11) is the density of the solid material and not the density of the stationary air. The mass fraction mf_{ϕ} is the mass fraction of ϕ in the surrounding solid, and not in the stationary air. In (6.11), we want

$$\rho D_{\phi} = \rho_{Air} D$$

so that $mf_{\phi} = mf_{v}$. Consequently, we must set D_{ϕ} to

$$D_{\phi} = \frac{\rho_{Air}}{\rho} D$$

where ρ_{Air} [kg m^{-3}] is the density of the air-vapour mixture, ρ is the density of the solid material, and D [m^2 s^{-1}] is the kinematic diffusivity of the vapour in stationary air, given by (6.10). We can take ρ_{Air} to be the density of dry air at $20°\text{C}$ (1.206 kg m^{-3}). The calculated mass fraction mf_{ϕ} is then the mass fraction of vapour in the stationary air (mf_{v} in Section 6.1).